How Cognitive Skill Development Impacts Job Market Outcomes: A Theoretical Analysis

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Abstract

This article analyzes the impact of different learning methodologies on the development of skills and their consequences in the labor market through a theoretical job search model. The paper presents an equation for modeling the dynamic acquisition of skills during education and applies it to a job search model with on-the-job training. The model is calibrated using data from the Program for the International Assessment of Adult Competencies (PIAAC), and explores the optimal design of the educational system with the goal of maximizing the aggregate match value in the labor market. The results indicate that a shift towards a greater emphasis on cognitive skills leads to improved labor market outcomes, including increased flexibility and mobility, and reduced skill polarization.

Keywords Education, Job Search, Mismatch, Learning, Skill polarisation.

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1 Introduction

To what extent the ability to adapt skills to different requirements is becoming important in a world of rapid technological advancement and labor demand changes? How should the educational system adapt? Can the government increase workers' flexibility through cognitive skill training during education?

The ability to adapt skills has become increasingly important in today's rapidly changing world with technological advancements and shifting labor demands. The continuous evolution of technology and computerization has dramatically altered the labor market in recent decades. This has resulted in a significant decline in the demand for routine jobs, regardless of whether these jobs are cognitive or manual in nature, and a corresponding rise in the demand for both high-skilled and low-skilled workers. This shift has been referred to as "job polarization" and has been extensively studied in academic literature, with a focus on the factors driving these changes and their impact on workers and wage inequality ((Acemoglu & Autor, 2011); (Katz & Autor, 1999); (Machin, 2003; McIntosh, 2002; Commander & Kollo, 2004)). The trend of job polarization is expected to persist, making it increasingly important for individuals to possess skills that are flexible and transferable to a variety of different roles and industries.

The impact of digitalization on the workforce is significant, affecting not just the volume and type of work, but also the way it is organized. An increasing number of tasks will be performed online and made tradable over the internet ((Dachs, 2018)). This means that companies and senior executives need to reevaluate their role in preparing workers for a rapidly changing economy by developing the necessary skills for new job demands ((Illanes, Lund, Mourshed, Rutherford, & Tyreman, 2018)).

As technology continues to evolve at a rapid pace, the ability to adapt to changing work and skills requirements is becoming increasingly important. This issue is addressed in the book "Anticipating and Preparing for Emerging Skills and Jobs" ((Panth & Maclean, 2020)). The authors argue that there is an urgent need to align education and training with the current and future needs of the economy, as the gap between acquired skills and labor market demands will only continue to widen.

New job opportunities will be created as technological, social, demographic, and political changes continue to accelerate, creating volatility and uncertainty. A future-oriented curriculum framework is required to enable learners to prepare and adapt to these changing needs with confidence. This requires a clear and effective action plan to match education and training systems with the rapidly evolving employment needs of each country.

Expert consensus indicates that the current gap between formal skills and competences and labor market needs will not only persist, but will widen in the future ((Redecker et al., 2010)). To effectively address these changes, an overhaul of the educational system, which was designed decades ago, is necessary.

This research proposes a novel framework to examine internal efficiency of education by integrating job search models with more traditional models of human capital, through the presence of skill accumulation during education. The model seeks to bridge the divide between educational skill development and labor market dynamics, aiming to elucidate the mechanisms through which educational policies shape employment outcomes in labor markets. The basis framework I propose is a theoretical job search model that exhibits on-the-job training, along the lines of (Postel-Vinay & Lise, 2020), with two major additions: the rate of on-the-job skill adjustment is endogenous and depends on a particular component of the two-dimensional skill

vector (cognitive skills), that is affected by the design of the educational system chosen by a theoretical planner. This setup requires to address two particular issues: identify which skill determines the rate of skill adjustment, and verify that this skill is actually trainable during education. Concerning the former, the literature in cognitive and learning psychology agrees that learning ability depends on cognitive skills. To cite (Jensen, 1989) <<review of evidence from psychological and educational literature on the relationship between individual differences in measures of learning and of intelligence suggests that no clear distinction can be made between the two cognitive processes>>.
Jensen concludes that both learning and intelligence reflect Spearman's g factor, which is thus related to the cognitive skill notion. Moreover,(Gathercole, Dunning, Holmes, & Norris, 2019) provides evidence that development of new routines depends on general cognitive resources (in particolar the so called "fluid intelligence").
The dependance between the rate of skill adjustment and cognitive skills will be validated and modeled using data from the italian subset of OECD PIAAC (Survey of Adults Skill), that will be also used to calibrate the whole model.
Regarding the trainability of cognitive skill, especially during the first stages of education, research in cognitive and learning psychology points to the fact that different methods of learning can lead to significant improvements in cognitive skills. For instance, inquiry-based learning has been found to lead to a better understanding and improved cognitive skills ((Guo, 2010); (Ismail & Elias, 2006)). Studies have also shown that cognitive skills can be improved through traditional methods such as memorization, practice, and problem-solving ((Carpenter, Just, & Shell, 1990); (Hegarty, 2004)). In summary, cognitive skills are trainable and can be improved through various methods of learning.
In Manghi and Bernardi(2022) !! we have analyzed how the study of logic in primary school helps to develop skills not only in mathematics, but also more generally. We provided empirical evidence designing an RCT that showed how an educational path based on an inquiry-based approach to the study of logic increased performance on a Raven matrix test for children. This RCT was registered in the American Economic Association of Randomized Control Trials, and is part of a larger research project involving other educational pathways characterized by an approach based on role playing and inquiry based learning in various areas of mathematics. The data collected in these projects supports the second assumption: that is, cognitive skills, and in particular the so-called "fluid intelligence", are trainable during the education. Therefore, changing programs and methodologies in the scholastic system, the government is able to strategically realign the initial distribution of multidimensional skills among individuals, placing a greater emphasis on cognitive abilities, at the expenses of practical tools and notions that will be needed to acquire marketable knowledge.
The search model is then used to describe formally the dynamics through which the development of different types of skills translates into a trade-off between flexibility and initial productivity, and to analyze the implications that a shift into flexibility has into wages' distribution and path, job mobility, value generated by the job market and capability of the economy to adapt to technological innovation and sectorial shifts.
First I propose an equation for skills dynamic during education. The functional form for such equation proposed is consistent with the literature, as in (Sanders, 2012), where the skill accumulation equation also depends on cognitive skill, but has a stochastic component. The functional form is also supported from the empirical literature in cognitive and learning psychology, in particular to (Vaci et al., 2019) In this longitudinal study that tracked the evolution of chess' ability depending on intelligence and practice, three main results are presented: more intelligent people benefit more from practice, linear returns from practice, diminishing returns from intelligence. The dynamics of skill accumulation are thus modeled through a Cobb-Douglas

function, which quantifies the gains in skill as a function of two key inputs: cognitive ability and practice. This formulation allows us to capture the multiplicative interaction between cognitive skills and practice in contributing to skill development, diminishing returns from cognitive skill, and linear returns from practice.

The skill accumulation equations define how the initial cognitive skill distribution in the individuals translates into an initial bidimensional skill distribution across workers, before their entrance in the job market, and therefore the initial distribution of match values and the rate of skill adjusment while working (on-the-job training). Given this latter feature, the stationary distribution of skills in the model also depends on the parameters of the skill accumulation equation for education, and the planner control variable.

The control variable is the stock of cognitive skill which is devoted to the accumulation of new cognitive skill. Different values of the government control variable define then a technological frontier (or a skill frontier) of possible stationary skills distributions. These parameters - together with the other parameters of the model - are calibrated in order to match the stationary skills and wages distributions with the skills and wages distributions observed in the PIAAC data. Estimating also the planner control within the parameters allows to assess the "actual" design of the educational system and compare it to the one that maximizes aggregate match value at the stationary distribution. In the job search model individuals are endowed with a multidimensional skill bundle that results from the educational system.

The learning process in the educational system can be seen as a transformation of their base cognitive skill level ("at birth") into a two-dimensional skill vector - that measure cognitive and applied marketable skills - dependent on a parameter chosen by the government. Changing this parameter practically corresponds to choosing the weights with which different skills are trained during education.

While entering in the job market they are randomly matched with firms, which have heterogeneous skill requirements along the same dimensions of the individual's bundles. When working, workers adjust their skills adapting to the requirements of the firm matched, producing an instantaneous flow of output (dependent from both the firm's requirements and the worker's skills) subject to a mismatch cost. They can receive outside offers with an exogenous arrival rate from other firms, in a Bertrand environment. The rate of adjustment is endogenous since it depends on the cognitive skills. This is the main difference between other similar job search models (Lindenlaub, 2014; Postel-Vinay & Lise, 2020; Sanders & Taber, 2012). By privileging the development of cognitive skills, individuals will be able to adapt more quickly to the requirements of the different firms, at the expenses of lower initial productivity. Therefore, the cognitive skill does not let the individual only to be able to respond to the cognitive skill requirements of the firm, but also allows him to adapt the other types of skills more quickly. This constitutes an additional indirect benefit of cognitive skills, which increases the flexibility of the worker and especially the capability to adapt to labor demand changes and technological innovation. The optimal solution of the model -i.e. the policy that maximizes aggregate value- is characterized by a shift of the initial skill vector towards cognitive skills. Cognitive skills allow to increase the aggregate match value favouring the speed of skill adjustment. Since higher cognitive skills increase the value of job offers with larger initial skill mismatch, the flexibility and the mobility in the job market will increase especially for low-skilled individuals. This increased capability of adapting to firm's requirements in a perfectly competitive environment decreases the transition costs for a worker (initial skill mismatch), increasing the value of wages, except for workers that have higher baseline cognitive skill, for which the gain in terms of an increased value of external offers is lower than the loss due to initial lower marketable skills. This is due to diminishing marginal returns of cognitive skill in the skill accumulation equation. Further developments of the research include a more detailed analysis of the implications on wages, an analysis on the implications on the capability to adapt to technological shocks in firm's requirements (sectorial shifts), and a more complex form for the production function.

The rest of the paper is organized as follow: in the next section I will illustrate the related literature, from the empirical and theoretical works about learning and the different learning methodologies, both from didactical mathematics and cognitive psychology, to the large literature about educational economics, human capital and job search models; then I will present the theoretical model and the dataset used for identification; at the end of the article I will present and compare simulations of the model both under the educational policy implied by the data and the policy that maximizes aggregate value, discussing the results and the relative implications.

2 Literature Review

This review navigates through interconnected domains of economic and educational theories, striving to amalgamate the rich insights from human capital theory, educational economics, labor market dynamics, and cognitive psychology.

Foundational to our understanding of educational investments is the Human Capital Theory, largely heralded by (Schultz, 1961) and (Becker, 1964). They articulate the intrinsic value of investing in human capabilities as a catalyst for economic expansion. Further down this line, the models of economic growth presented by (Romer, 1990) and (Lucas, 1988), underpin the influential role of knowledge and skill accumulation on economic development, emphasizing the pivotal impact of human capital on proliferating innovation and growth. While the presence of skill accumulation during education resemble Lucas model, it is important to underline that Romer and Lucas cast light on the macroeconomic implications of skill accumulation and investment in human capital, seen as amount of resources allocated in the education (external efficiency), while my exploration investigates the optimal allocation of given resources within the educational system (internal efficiency) and its subsequent micro-level outcomes in the labor market.
The notion of "internal efficiency" in education explores the dialectic between educational inputs and their resulting outputs, primarily in the form of acquired skills and knowledge (Coleman, 1968). Studies such as (Dearden, McIntosh, Myck, & Vignoles, 2002) and (Heckman, 2007) explore the economic returns to education, whereas our inquiry delves into the nuanced mechanisms through which educational investments transmute into cognitive skills and competencies.

The theoretical part with the job search model, it is inserted directly into the literature that studies the allocation of multidimensional skill bundles and the determinants of wage dispersion through theoretical search models, with the difference that my focus is on the impact of educational policy on different job market outcomes, more than the determinants of wage dispersion. A new emphasis on the roles of both quantity and quality of human capital in the development process, moreover, has given policy makers new appreciation of the importance of education–labor market linkages. The role of the quality has been much less studied than employment outcomes, particularly in developing countries, and is thus less understood. Perhaps degrees attained by young people have greater weight during the school to-work transition ((Allen & van der Velden, 2007), whereas skills and knowledge prove more important in the long term. However, if the skills acquired in education relate to a very specific occupation, technological change could make these obsolete. The focus on technological development that increases the complexity of jobs has been highlighted in Machin and Reenen (1998) and Wolff (2000). In particular, they highlight the increasing speed in the change of the skill demanded in the job market. Attempts to model the allocation and pricing of heterogeneous supply and demand of indivisible and multi-dimensional bundles dates back at least to Tinbergen (1956) and the hedonic model of Rosen (1974). Recently (Lindenlaub, 2014) estimates the quadratic-normal assignment model of Tinbergen (1956) along two dimensions of skills (manual and cognitive) for two different cohorts using a combination of O*NET and NSLY data to estimate the distribution of skill requirements conditional on worker's skill bundle. She finds an interesting pattern of technological change: the complementarity between her measures of cognitive worker skills and cognitive job skill requirements increased substantially during the 1990s, while the complementarity between manual job and worker attributes decreased. She then analyzes the consequences of that technological shift for sorting and wage inequality. The same dataset is used also in Lise and Postel-Vinay, a structural model of on-the-job search in which workers differ in skills along several dimensions (cognitive, manual, interpersonal...) and sort themselves into jobs with heterogeneous skill requirements along those same dimensions. Using the above mentioned dataset, the authors used their model to shed light on the origins and costs of mismatch along the cognitive, manual, and interpersonal skill dimensions. The presence of skill accumulation and learning during education is also similar to (Sanders, 2012), in which the is a two dimensional skill vector of manual and cognitive skills that updates during education with a stochastic component. Other two recent papers are particularly related. Taber and Vejlin (2020) estimate a model which allows for search, human capital accumulation and non wage amenities. Workers are modeled as having a time invariant relative ability at each job-type in the economy. In the absence of frictions they would choose a single job-type and remain indefinitely. Human capital is assumed to be general and accumulated while working. Job mobility is informative about the degree of search frictions, and wage cuts are informative about non wage amenities. Taber and Vejlin (2016) model relative ability between jobs/occupations as an unobserved vector with dimension equal to the number of job-types in the economy. The aim of my work is to use these setups to study the implications of the choice of the internal composition of the educational programs, presenting a new framework to analyze educational strategies in terms of job market outcomes.

In understanding the translation of educational outcomes to labor market viability, the comparison by (Hanushek, Schwerdt, Woessmann, & Zhang, 2017) between general and vocational education provides valuable insights into the life-cycle labor-market outcomes, further propelling our scrutiny into the relevance and application of skillsets within the labor markets. Similarly, (Lamo, Messina, & Wasmer, 2011) illuminates the potential friction and adaptabilities presented by specific skills in the labor market, a concept that is deeply embedded in our exploration of educational efficiencies.

A deeper dive into labor market dynamics by (Guvenen, Kuruscu, Tanaka, & Wiczer, 2020) elucidates the multifaceted nature of skill mismatch, providing a vital perspective that enriches our examination of educational strategies and their aptitude to navigate such skill discrepancies. Furthermore, (Flinn, Gemici, & Laufer, 2017) offers a pivotal foundation for understanding the mechanisms of job search, skill matching, and training in the labor market, accentuating the necessity of correlating educational outputs with labor market demands and aligning skill acquisition with industry requisites.

Enriching this analytical approach towards skill acquisition are the insights from cognitive psychology, particularly by (Vaci et al., 2019), which unpack the synergies between intelligence and practice in bolstering skill development. These insights propel our understanding of optimizing cognitive skill development within educational structures and paradigms.

This paper seeks to stitch these varied literatures into a unified analytical framework, intertwining the cognitive development of skills and multi-dimensional labor market dynamics.

3 Search Model

The main feature of the framework proposed is to integrate a dynamic of skill accumulation during education to a job search model with multidimensional human capital. The goal is to study the implications that different educational policies (characterized by the stock of cognitive skill that is allocated to the accumulation of different components of the skill vector) have on the mobility in the labor market and on the flexibility of workers in adapting their skills in response to market demand changes and technological development. The environment of the job search model is similar to Postel-Vinay and Lise (2020) and Postel–Vinay and Robin (2002).

3.1 Environment

The model is made by a continuous of agents endowed with a two-dimensional skill bundle $x_0 = (x_{0C}, x_{0A})$, which take values in the set of possible skills $X = (X_C \times X_A) \subset R^2$, as in Postel-Vinay and Lise (2020).
x_C represents cognitive skills, while x_A represents applied skills. With cognitive skills, similarly to the other job search models, I refer to what is broadly defined in cognitive psychology literature as "fluid intelligence", that mental capability which depends minimally on prior learning and consists in the ability to formulate abstract mental models to be applied in different circumstances, and that therefore it allows to understand different contexts more easily. By applied skill, on the other hand, I refer to the set of tools, acquired notions that translate into marketable knowledge, and that are directly applied in the labor market and used for production. Knowing how to draw up a balance sheet, how to use a particular econometric technique, or how to make a table all fall within the broad concept of applied skill, even if some of this tasks are "mental activities" and a different level of cognitive skill is necessary to acquire those capabilities. Cognitive skills are then used to accumulate applied skills, that are used in production.

In the future, it is possible to extend the model allowing for more dimensions of applied skills (for example, to separate manual skills) in order to analyze more deeply some phenomena such as job polarisation, but for the purposes of this article I will keep a simpler bi-dimensional framework, since the main focus is on the role of cognitive skills as "technology" in the accumulation of the skills that are used in production, and the trade-off between how much to improve this "technology" by training cognitive skills and the accumulation of applied skills before entrance in the job market. The use of a similar two-dimensional skill vector is present also in Lindenlaub and Postel-Vinay (2016) and Lindenlaub (2017). Individuals accumulate their skills during the educational period at a first glance, and then they enter in the job market. While working, they will adjust their skill to the requirements of the firm matched. The time in which they enter in the job market is an exogenous parameter and will be denoted by t_1. It is exogenous since the focus of the article is not to analyze heterogeneity in educational choices, and moreover the time of compulsory education is fixed. Government may decide to change it, so possible extensions of the model can consider t_1 as an additional policy variable. Once workers are in the job market, they are matched with a firm whose requirements y are drawn from the sample distribution $H(y)$ which take values in the set of possible skill vectors $X \subset R^2$.

When they are matched with a firm, they produce an instantaneous flow of output $p(x, y)$ and adjust their applied skill x_A to the firm requirement as in other search models with on-the-job-training, with the difference that the rate of adjustment will depend on cognitive skills, that will affect the value of the match even if only applied skills are directly used in production. Cognitive skills level will be determined after the first stage of skill accumulation during the educational period. The assumption that cognitive skill is not increased after the educational period is supported by the extensive evidence showing a slow decline-that can at most be stopped- in cognitive skill after the age of 20 (de Chastelaine, Wang, Minton, Muftuler, & Rugg, 2011; Morrison & Hof, 1997; Persson et al., 2006; Salat et al., 2004). I considered this assumption quite realistic both given this empirical evidence and the context of this model. Given the definition of applied and cognitive skills, firms do not have requirements in term of cognitive skills, so there won't be any skill adjustment for them. It is not common to hear about firms asking for minimum IQ levels as requirements. Maybe they can ask for competencies that are difficult to acquire in terms of cognitive skills effort, and thus more easily acquired for individual with high intelligence, but this is in line with the interpretation of this model of cognitive skills being the "technology" in learning.

The equation for skill accumulation during education depends on initial cognitive skills and practice. As mentioned in the introduction, findings in the cognitive/learning psychology literature ((Vaci et al., 2019)) demonstrated a joint effect of intelligence and practice - i.e. that the most intelligent benefit most from the practice - they also estimated the returns from each of the two inputs, also restricting the analysis to the sub-sample considered. The evidence gathered by the authors suggests diminishing returns for intelligence, and constant returns for practice. A similar formulation, with diminishing returns from skill accumulation and linear returns from practice, is present also in ((Sanders, 2012)), but with a stochastic component. The skill accumulation equation proposed in this work can be seen as a deterministic version of the latter. In this model, "practice" is interpreted as the share of the stock of cognitive skills allocated at each time in the accumulation of cognitive or applied skills (denoted by $s_i(t)$), and it is the policy variable. Being x_i, $i = A, C$ respectively applied and the cognitive skill, the functional form for the learning equation is as follows:

$$\dot{x}_i(t) = x_C(t)^{\alpha} s_i(t) \tag{1}$$

In practice, the skill change in one period of time is a Cobb-Douglas with initial cognitive skill and amount of specific practice for that skill in that period as inputs. The equation above is calibrated with the PIAAC data as explained in the Data and Identification section.

As previously defined, $s_C(t)$ (and $s_A(t) = 1 - s_C(t)$) are the weights according to which cognitive and applied skills are trained during education, so that $s_C(t)$ is the policy variable. So, $s_C : R^+ \to [0,1]$ is the control of the government, and $S = \{s : R^+ \to [0,1] | s \in L^\infty[0,1]\}$ the set of possible controls.
Thus, $x^i(x_0^i, y, s_C, t)$ is a two dimensional vector representing the value of the skills at time t given the initial level of cognitive skills x_0^i, the requirement of the matched firm y, and the choice of the government for the policy variable s_C. The initial value of applied skills is zero since they are, by definition, acquired from the external during education and work.
Since the inputs (x_0, y) are random variables, defined in the first paragraph, $x^i(x_0^i, y, s_C, t)$ is the composition of a deterministic function with random variables and is thus a stochastic process. To simplify the notation in this part I will denote the trajectory $x^i(x_0, y, s_C\cdot) : R^+ \to R^2$ obtained for a specific realization of the random variables (x_0, y), and a specific choice of s_C ,simply as $x^i(t)$.

This trajectory, during the period of education, so for $0 \leq t \leq t_1$ solves:

$$\begin{cases} \dot{x_C^i}(t) = x_C^i(t)^{\alpha} s_C(t), & x_C^i(0) = x_0^i \\ \dot{x_a^i} = x_C^i(t)^{\alpha} s_A(t), & x_A^i(0) = 0 \end{cases} \tag{2}$$

The parameter α is calibrated matching model skills ditribution with the sample skills distribution in the PIAAC data.

The law for the period of work is similar to Postel-Vinay and Lise (2020) and other models of on-the-job training, so that workers adjust skill linearly with respect to firm requirements, but the rate of skill adjustment depends on the cognitive skill (in other cited models is an exogenous parameter).

In this period (from t_1) skills are adjusted according to the following ODE (ordinary differential equation):

$$\begin{cases} \dot{x}^i_C(t) = 0 \\ \dot{x}^i_A(t) = x^i_C(t_1)^\gamma \max(y - x_A, 0) \end{cases} \tag{3}$$

This relation between rate of adjustment and cognitive skills is assessed using PIAAC data, as will be explained more in detail in the next section together with the rest of the identification strategy, where the above relation is estimated. Assuming a linear adjustment of worker skills in response to firm requirements provides a simplified yet effective representation for several reasons. First, it offers a straightforward and intuitive foundation for modeling, facilitating interpretation and analysis. Second, many job training programs historically adopt a linear progression, indicating that workers often develop skills in a predictable, incremental manner. Lastly, from an economic perspective, firms tend to favor consistent and clear skill trajectories for efficiency, making a linear assumption a reasonable starting point. While this linear framework may not capture every nuance, it serves as a robust initial approximation in understanding worker-firm dynamics and is therefore assumed also in other job search models.

Both equations admits a closed form solution, in particular (3) can be solved separating variables in the first equation and then solving it by direct integration,and then substituting in the second equation that can then be solved by direct integration. (4), indeed, is constant when $x_A \geq y$ and can be solved directly applying the standard general integration formula for linear ODE's when $x_a \leq y$. The derivation of these solutions will be explained more in detail in the appendix. Consequently, such closed form solutions I can write the specification for the stochastic process x^i as :

$$x^i(x_0, y, s_C, t) = \begin{cases} ((x^i_0)^{1-\alpha} + S_C(t))^{\frac{1}{1-\alpha}} \\ \int_0^t (x^i_C(s))^\alpha s_A(s) ds \end{cases} \quad 0 \leq t \leq t_1 \begin{cases} x^i_C(t_1) \\ \max(x^i_A(t_1), (y - e^{x^i_C(t_1)^\gamma (t-t_1)}(y - x^i_A(t_1))) \end{cases} \quad t_1 \leq t \tag{4}$$

Where $S_A(t) = \int_0^t s_A(s)ds$ and $S_C(t) = \int_0^t s_C(s)ds$.
Now consider the dynamic of x_A in $[0, t_1]$. First, recalling that $s_A(t) = 1 - s_C(t)$ the above specification can be rewritten (for the interval $[0, t_1]$) as:

$$x^i_A(t) = \int_0^t (x^i_C(s))^\alpha ds - \int_0^t x^i_C(s)^\alpha s_C(s) ds \tag{5}$$

Where the last term is equal to $x_C(t) - x_{0C}$ from (3), while the term in the middle, again by (3) is the value of $x_C(t)$ in the case in which the choice for $s_C(t)$ is $s_C(t) = 1$ for all t in $[0, t_1]$, that is the maximum possible choice for s_C. Always according to (3), in this case we would have $S_C(t) = t_1$ and thus $x_C(t) = (x^i_{0C})^{1-\alpha} + t_1)^{\frac{1}{1-\alpha}}$. Then, we can rewrite (7) as:

$$x^i_A(t) = (x^i_0)^{1-\alpha} + t_1)^{\frac{1}{1-\alpha}} - x_0 - (x_C(t) - x_0) = (x^i_0)^{1-\alpha} + t_1)^{\frac{1}{1-\alpha}} - x^i_C(t) \tag{6}$$

Given the initial bundle of skills x_0, the skill bundle at the end of the educational period - at time t_1- is deterministic as $x(t_1)$ can be computed as in (6) once the planner has chosen its control.

Consider so the "education" map $J : S \times X \to X$, defined as

$$J(s_C, x) = \begin{cases} J_C(s_C, x_C) \\ J_A(s_C, x_A) \end{cases} = \begin{cases} ((x_0^i)^{1-\alpha} + S_C(t_1))^{\frac{1}{1-\alpha}} \\ ((x_0^i)^{1-\alpha} + t_1)^{\frac{1}{1-\alpha}} - x_C^i(t_1) \end{cases} \tag{7}$$

J maps the initial cognitive skills distribution of the population into a bi-dimensional skills distribution of workers at the end of the educational period.

Since the action of the government is relevant in the model only because of the values determined for the skill bundle at t_1 we can see from (9) that the government choice will affect individual skill's evolution only through the term $S_C(t_1)$. We can therefore assume that the government is not choosing a control in a set of functions, because it is enough for it to determine a real number $S_C(t_1)$ in $[0, t_1]$. This will simplify a lot the optimization problem that I will present in the next section, because it will be a one-variable optimization problem instead of a dynamic programming problem. So we can consider the function J as a function of the government's control $S_C(t_1)$ and of the initial skill bundle, that from now on we will denote simply by s.

So J defines a frontier for the initial skill distribution of workers, with any point of the frontier being a different initial distribution of skills depending on the educational design chosen by the government.

We can adopt the substitution $t' = t - t_1$ and consider as starting time the time of entry in the job market. In this way, the skill process will follow (5) and the initial condition will be given by $J(s, x_0)$. I will without loss of generality normalize time and assume $t_1 = 1$. So adopting the above mentioned shift in time and using the closed form solution in (8) we can write the process for the evolution of individual i skill bundle from the time of entry in the job market (which is now time 0) as:

$$x^i(s,t) = \begin{cases} J_C^i(s) \\ \max(J_A^i(s), (y - e^{(J_C^i(s)(t)^\gamma}(y - J_A^i(s))) \end{cases} \tag{8}$$

Where:

$$J^i(s) = \begin{cases} J_C^i(s) \\ J_A^i(s) \end{cases} = \begin{cases} (x_0^{i\,1-\alpha} + s)^{\frac{1}{1-\alpha}} \\ (x_{0C}^{i\,1-\alpha} + 1)^{\frac{1}{1-\alpha}} - (x_0^{i\,1-\alpha} + s)^{\frac{1}{1-\alpha}} \end{cases} \tag{9}$$

This defines the set of possible initial skill distributions of workers. The simulation of the model converges to a stationary distribution that is sensible to the initial skills distribution, given the endogeneity of cognitive skills in the process of on-the-job-training that affects match dynamics and wage trajectories. Then the government policy variable defines a technological frontier of possible stationary skill distributions. The actual point of the frontier and the other parameter of the equations in the model will be identified using PIAAC dataset -matching stationary distributions of skills with sampling distributions -and compared with the optimal point that maximizes the aggregate match value in the job market.

3.2 Production Function and Value of the match

As explained above, once paired with a firm the worker will produce an instant flow of output that will be directly proportional to both the skills of the individual and the requirements of the firm (so that the firms with the highest requirements are the most productive), but that will be penalized by a mismatch cost. Hence the form for $p(x,y)$ is:

$$p(x,y) = f(x,y) - c(x,y) \tag{10}$$

Where

$$f(x,y) = x_A y_A \tag{11}$$

while the mismatch cost

$$c(x,y) = k_A \max(y_A - x_A, 0)^2 \tag{12}$$

So that the cost mismatch will arise only if the worker is under-qualified. As previously discussed, applied skills include all the competencies that are used in production, and cognitive skills is the technology needed to acquire those competencies, so the output flow will depend (directly) only on applied skills. The simple shape of this dependency is chosen to simplify the tractability and the possibility of analyzing a closed form solution. Hypothesizing a more complex functional form, perhaps with a non-linear dependence on job's skill requirement, could maybe be more realistic but would have little impact on the model given that this does not aim to analyze dynamics linked to the level of production in absolute terms, but more concerning the allocation of different skill bundles and the trade-off between cognitive and applied skills. As long as the production function depends only on applied skills, therefore, the core of these trade-offs is not altered. While working, individuals meet other firms with instantaneous probability λ. The transition rate λ is exogenous and retrieved from the PIAAC dataset. Firms compete à la Bertrand, workers and firms are risk neutral and have discounting rate β. Let P(x,y) be the total value of the match between an individual with skill bundle x matched with a firm with requirements y. If we assume for simplicity that unemployment flow is zero, then the value of being unemployed is just zero and P(x,y) is the surplus value from the match. The worker's value from a match is given by W, where $W \leq P(x,y)$ (otherwise the firm will not be in the match) and $W \geq 0$ (otherwise the worker will be better of quitting into unemployment. This surplus will be shared between the firm and the worker according to the sequential auction model as in Lindenlaub (2017) and Postel-Vinay and Lise (2020). In the sequential auction model, firms offer take-it-or-leave-it wage contracts to workers. When a worker receives an outside offer,the current and outside employers Bertrand-compete for the worker. So, if a worker currently in a match valued P(x,y) with a type-y firm gets an outside offer from a type-y' firm, whose match value would be P(x,y'), we have three cases:

- $P(x,y') \geq P(x,y)$, and the worker accept the offer becoming employed with type-y' firm with wage value W=P(x,y);
- $P(x,y) \geq P(x,y') \geq W$, and the worker stays in the initial match rebargaining the wage to a value W=P(x,y');
- $P(x,y') \leq W$, and the worker stays in the initial match without changing its wage value.

It follows that the match value P(x,y) solves:

$$\beta P(x,y) = p(x,y) - \mu P(x,y) + \nabla_x P(x,y)\dot{x}(x,y) \tag{13}$$

That is, the annuity value of the match is equals to the output flow $p(x,y)$, minus the expected loss from job destruction (rate μ), plus the increase in value due to skill adjustment, in which the marginal change of the skill vector is given by (5). Notice that the match value is independent by the expected value of future job offers. Future job offers will only affect the sharing of match surplus. This is a direct implication of Bertrand competition. If a worker gets an outside offer with higher value, in fact, it will change job leaving a vacancy worth zero and taking all the value from the previous match, as argued above. If the match value of the outside offer is indeed less than P(x,y), the worker will stay in the match. In any case, the continuation value of the match is still P(x,y). The partial differential equation (PDE) (13) admits a closed-form solution which is given by:

$$P(x,y) = \frac{y_A(x_A + x_C^{\gamma}(y_A - x_A))}{\beta + \mu} - \frac{k_A(y_A - x_A)^2}{\beta + \mu + x_C^{\gamma}} \tag{14}$$

In the case in which $x_A \leq y_A$ and $x_C \leq y_C$. If one (or both) of this inequalities is not satisfied, and so the worker is overqualified or perfectly qualified, the mismatch cost will just be zero and we'll have $P(x,y) = x_A y_A$, since there will not be on-the-job training. The derivation of this closed-form solution is explained more in detail in the appendix.

Let's look at the match value (14): the first term represents the total value from applied skill's output flows, which takes into account a progressive adjustment of a worker's skill toward firm's requirement, while applied skill's mismatch cost (last term) is also discounted for the rate of adjustment (x_C^{γ}), which progressively reduces the mismatch. Higher firm requirements will provide higher match value only if the worker has enough cognitive skills: the first term will increase with the mismatch to an amount proportional to x_C, while the mismatch cost will increase (so the second term of the match value will decrease). Whether the increase in the value of future output flows offsets the increase in initial mismatch will depend on cognitive skills level. This will prevent low skilled individuals to join high requirement firms. Cognitive skills then affect the match value through the progressive reduction of the mismatch. Equation (14) clearly shows how cognitive skills will be more important in a given match value when the mismatch is high, making cognitive skills training more valuable for individuals that face on average higher mismatch (low and medium skilled individuals). This latter feature comes from linearity of skill adjustment with respect to job specific requirements. Dropping this assumption - for example assuming that skill adjustment is independent from the current firm/job and the size of the mismatch- will reduce this dynamic for which cognitive skill training is relatively more valuable for medium and low skilled individuals, but not completely, given diminishing marginal returns of cognitive skills in skill accumulation.

3.3 Objective function and Match Distribution

Once an individual is matched with a firm, it produces an instantaneous flow of output $p(x,y)$, whose lifetime value $P(x,y)$ is derived in the previous section. As argued before, the private value of the match does not depend on the probability of future job offers, due to Bertrand competition. Once workers are matched with a type-y firm , the match value of worker i is then given by:

$$P^i(J(s, x_0^i), y)$$

Where I recall $J(s,x_0)$ is the skill bundle of the worker with initial cognitive skill x_0 after the educational period conducted with the policy s. Thus, we can compute the expected value generated by the individual i with the initial skill bundle x_0 taking the expectation with respect to firm requirements:

$$EP^i(s,x_0^i) = E_X[P^i(J(s,x_0^i),y)] = \int_X P^i((J(s,x_0^i),y)q(y)dy \tag{15}$$

Where q is the density of y given x_0, i.e. the probability for the worker with initial skill vector $J(s,x_0^i)$ of being matched with type-y firm. Since EP^i varies over individuals due to their different initial skill bundle which characterize the individual, we can consider EP^i as a function of the initial skill bundle, defined as $EP(x_0^i,s) = EP^i(s)$ The aggregate value produced by the job market integrating $EP(x,s)$ over the set X, given the density $g(x,s)$ of the stationary skills distribution:

$$P(s) = \int_X [EP(x,s)]g(x) = \int_X P^i(J(s,x_0),y)q(y)g(x,s)dxdy \tag{16}$$

So that the dynamic optimization problem that the government has to solve is the following:

$$\max_{s\in[0,1]} P(s) \tag{17}$$

In order to solve this optimization problem, densities $g(x,s)$ and $q(y)$ are needed. The initial distribution for cognitive skills is chosen accordingly to IQ distribution in population, which is largely recognized to be a normal distribution. Then applied skill is determined with the skill accumulation equation for education, once educational policy is given. Depending on the educational policy, there will be different initial skills distribution and therefore the model will converge to a different stationary distribution. For any value of s, density $g(x,s)$ is then the stationary skills distribution at which the model converges. The value of s that maximizes aggregate match value at the stationary distribution will be compared with the level of s (and thus the skills distribution and total match value) implied by the data. Distribution of firms' requirements, $q(y)$, is retrieved from the same PIAAC dataset. The optimization problem has been solved numerically and will be discussed at the end of the next section, but from the analytical form of the objective function it is possible to assess some implication. Considering equation (16) and substituting for the match value, we obtain:

$$P(s) = \int_X \left(\frac{y_A(x_A + x_C^\gamma(y_A - x_A))}{\beta+\mu} - \frac{k_A(y_A - x_A)^2}{\beta+\mu+x_C^\gamma}\right)q(y)g(x,s)dxdy \tag{18}$$

From the map J that defines the skill frontier at the end of education -equation (9)- it follows that increasing s will increase initial cognitive skills and decrease initial applied skills. However, the expression for the match value (14) shows how the overall impact for a worker will depend on the size of the mismatch he is facing. The higher is the average mismatch, the more the decrease in applied skills will be compensated by the increase in the rate of adjustment. Moreover reduction of mismatches will allow low and medium skilled workers to work for higher requirement firms, and thus, given the dynamic of skill evolution given in equation (9), their applied skills will converge to higher levels at the stationary distribution. This dynamic, together with diminishing return of cognitive skills in skill accumulation (equation (1)) makes an increase in s to change the stationary distribution $g(x,s)$ towards higher levels of both cognitive and applied skills for initially less gifted individuals, and towards slightly higher levels of cognitive skills but lower levels of applied skills for more talented individuals. So it will reduce dispersion in the stationary distributions of skills. An increase in s has the negative effect of reducing skill level and productivity for highly skilled individuals, and initial productivity of all the workers. But it also has the positive effect of allowing low and medium skilled individuals to work for more firms (increasing relative value of outside offers with greater mismatch) and thus being able to increase their skills to higher levels, thanks to higher average cognitive skills (and thus flexibility).

4 Data and Identification Strategy

4.1 Dataset

The dataset used is the Italian subset of the Survey of Adult Skills, conducted as part of the Programme for the International Assessment of Adult Competencies (PIAAC). This international survey is conducted in over 40 countries/economies and measures the key cognitive and workplace skills needed for individuals to participate in society. The Italian survey that I used consisted of 2209 observations (after cleaning for missing values), with questions on the workplace, job status, job history, actual job's requirements, skill mismatch and competencies in different tasks and cognitive skills test (both for literacy and numeracy). I used this data to estimate the job transition rate, the firms requirements distribution, the equations for skill accumulation both during education and on-the-job and the technological frontier on workers' skill in which the government acts its control. First I summarize how I measured the main variables from the dataset:

- Cognitive Skill. In order to measure x_C, I took the average result of the verbal and numerical reasoning tests performed by the individual during the survey.
- Cognitive Skill Requirements. To measure the cognitive skill requirements of firms, I used questions regarding the quantity and difficulty of cognitive skill tasks performed at work.
- Applied Skill Requirements. In order to assess the applied skill requirements, I used ISCO (International Classification of Occupations skill) level classification. In the ISCO context, skill is defined as the ability to carry out the tasks and duties of a given job, so it fits the definition that I gave of Applied Skills.
- Applied Skill. Once a grid for possible applied skills level is defined using ISCO categories, Applied Skill is determined as follows: exploiting questions about competences mismatch, and given the ISCO classification of an individual's job, the individual Applied Skill is determined as equal to the firm requiremnt if the worker is perfectly matched; one level below if the worker is underqualified; two levels below if the worker is seriously underqualified; and one level above if the worker is overqualified.
- Rate of Skill adjustment. The rate of skill adjustment was measured using a particular Index constructed by PIAAC, "Index of Learning at work" which measure learning new things from supervisors or co-workers; learning-by-doing; and keeping up-to-date with new products or services.
- For the job transition rate I estimated the yearly rate of job change from the history of previous jobs.

Once I obtain measurements for the above variables, matching the stationary distribution for skills in the model with the sampling distributions observed in the data allows to estimate the parameter of the education skill accumulation function and the implied government control value; while studying the correlation in the data between cognitive skills and learning index I modeled the equation for on-the-job training.

4.2 Estimation

Considering equations (8) and equation (9), assuming an ex-ante normal distribution for cognitive skills, we have that for any given α and s we will converge to a different stationary distribution for applied and cognitive skills in the job market. Having derived a sampling distribution for these skills from the data presented above, I was able to calibrate numerically α and s in the way that better fits the data.

To calibrate α and s, a grid search approach was employed. This technique involves systematically exploring a predefined range of potential values for both parameters, evaluating stationary distributions implied by each combination of (α, s) against observed distributions of skills. For each combination, the model generates distributions which are compared to empirical data. Then, the combination (α, s) that provides the least sum of squared errors (summing squared errors for both skills dimension) is chosen:

$$(\alpha, s) = argmin \sum_i (x_A^i data - x_A^i(\alpha, s))^2 + (x_C^i data - x_C^i(\alpha, s))^2 \tag{19}$$

The range of potential values is (0,1) for both the variables. For s it comes from its definition as explained in 3.1, while for α it is the set of possible values consistent with dimishing returns of intelligence in learning and so with the empirical literature cited in this article. The results of the calibration are summarized in the table below, where the Average Squared Error (ASE) is reported as a measure of goodness of fit.

alpha	s	ASE
0,17	0,22	0,0205

In this way, I calibrated the coefficient α for skill accumulation during education, and I measured the actual policy implied by the data, in order to compare it with the optimal policy that maximizes aggregate value. The skills distribution under the actual policy is reported below:

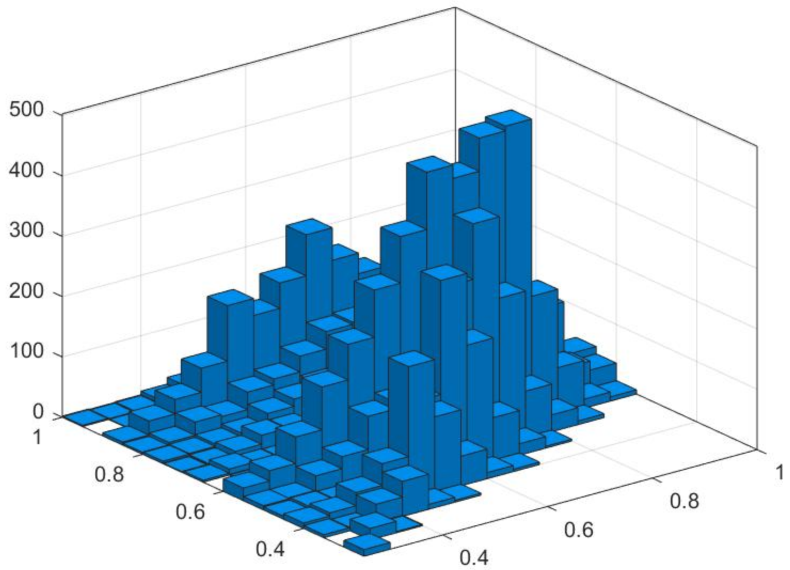

Figure 1: Joint skill stationary distribution for s=0.22, $\alpha = 0.17$

4.3 Estimation of the rate of skill adjustment

In equation (3) I considered the rate of skill adjustment to be equal to x_C^{γ}. To validate this functional form and estimate γ, I modeled the relationship between rate of learning on the job and cognitive skills using the corresponding variables in the dataset. Given z, the rate of skill adjustment measured with the "Index of Learning at work", I conducted the following regression to estimate (3):

$$\log(z) = \gamma \log(x_C) \tag{20}$$

Obtaining the following results:

	log IQ
Coeff	0,36***
S.E.	0.11
p-value	0.002

The results provide a measure for the gamma coefficient to be used in simulating the model and are in line with the aforementioned empirical evidence on the joint effect of intelligence and practice on skill development.

4.4 Simulation algorithm and Results

The model solution and simulation has been run on Matlab. The parameters have been chosen accordingly to the previous section, as the coefficient α for the educational map (9) and the coefficient γ for the rate of skill adjustment, as previously discussed. Starting from a cohort of 2209 agents, with normally distributed initial cognitive skill, and given the government's choice s, (9) is used to compute the initial skill bundle. Then, for every individual, using the closed form solution (14), it is possible to compute the match value of the individual for any firm type, and thus to take its expectation with respect to firms requirements. Then, integrating over all the individuals, (16), $P(s)$ is obtained. Then, $P(s)$ is computed for any possible value of s in a grid from 0 to 1, and the maximizer is chosen. A simulation of the model is not needed in order to find the solution, but I have simulated it up to convergence to a stationary distribution for wages and skills, both with the optimal control and the control estimated in the previous section to match the data ($s = 0.22$), in order to study the dynamics induced by the optimal choice. The optimal choice for the planner given by the algorithm corresponds to $S = 0.3878$, so a significant increase in the practice weight for cognitive skill.

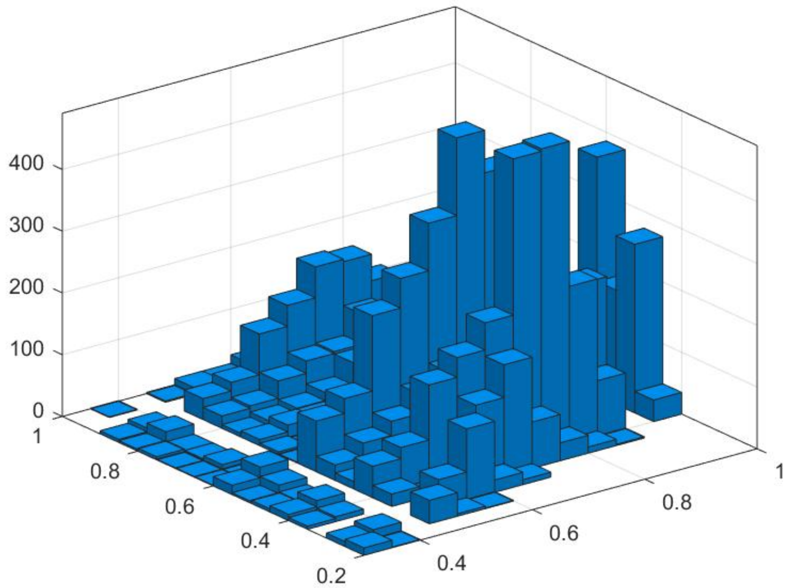

Figure 2: Joint skill stationary distribution for the optimal planner choice s=0.3878

As we can see, the overall distribution is shifted towards higher values of cognitive skill. However, the proportion of high cognitive-skill individuals which have medium-low levels of applied skill significantly increases. Due to the initial lower applied skill, indeed, more individuals are prevented from choosing top firms in terms of applied skill requirements, due to an higher initial mismatch cost. This prevents adjustment to top applied skill levels for many gifted individuals. On the other hand, the average applied skill for medium and lower cognitive skills individuals is higher. Of course, increasing initial cognitive skills facilitates the acquisition of future skills, to the detriment of the initial applied skills. The individual will therefore have less specific knowledge (and will be less productive initially) but at the same time will be faster in learning. If the burden in learning cognitive skills during education is too high, the lack of initial productivity will be too high to be compensated for by its future increase. Given the diminishing returns of cognitive skill in learning, this increase in general cognitive skill has major benefit for less skilled workers. Their mobility is increased and so is the ability to switch to more productive firms. For the most skilled workers the loss in initial productivity is not compensated by the lower gain they have in terms of increased cognitive skill (diminishing returns), and thus the match value with top firms will be more affected by the decrease in their initial applied skill than by the slight increase of their cognitive skill. For this reason, as pointed above, they may be in a position to not accept matches with the most productive firms. Moreover, since the probability of encountering them is exogenous, a high cognitive skill does not increase the chances of finishing in a good match. There are therefore two effects of the increase in cognitive skills. On the one hand, initial productivity is sacrificed for higher productivity in the future, and this applies to all types of workers. On the other hand, job mobility and the quality of future matches (and therefore their future applied skill) only increase for workers with medium-low skills. It should be specified that an increase in cognitive skill allows even initially less gifted workers to be able to work in firms with medium-high requirements. In fact, if a worker with a relatively low-medium skill encountered a productive firm, with low cognitive skill policies the mismatch value would be too high, and the individual would not be able to work for that firm. With a cognitive skill development policy, on the other hand, a higher percentage of workers can potentially work for more productive (or important) firms. The initial specific skills, in fact, are in both cases irrelevant for the value of the match which becomes constituted above all by the expectation of future productivity once the worker has acquired the necessary skills and by the mismatch value. Therefore, marginal differences in initial applied skills, for less skilled individuals, are less relevant compared to an increase of the learning rate under the optimal policy. Referring to the match value (16), in particular to numerator in the first term capturing the output flow:

$$y_A(x_A + x_C^{\gamma}(y_A - x_A))$$

when the worker is strongly under skilled, the first addend is not very relevant compared to the second, therefore an increase in cognitive skill is more valuable and can change the value of the match in a decisive way.

Given the diminishing returns of cognitive skill in learning, an increase in general cognitive skill benefits less skilled workers more. As argued, their mobility is increased and so is the ability to switch to more productive firms. An important result, which will be deepened in the future developments of this article, is the general increase in resilience to shocks in the requirements of the firms, and therefore a greater adaptability to technological change. I report the paths for average productivity resulted from the model simulations, conducted both with the planner's optimal policy and the actual educational policy.

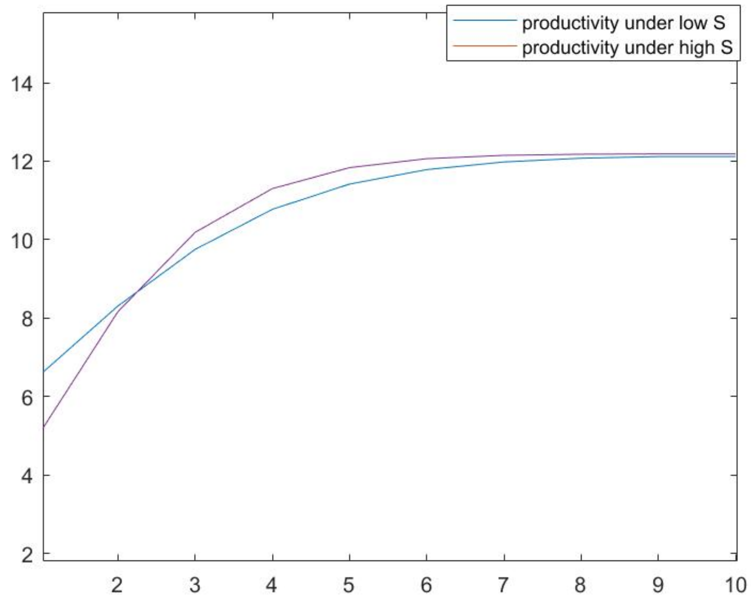

Figure 3: Average productivity under the optimal control is initially lower, but it grows faster thanks to the higher rate of skill adjustment

Establishing a higher practice weight for cognitive skills means reducing the amount of training in applied skills, therefore sacrificing initial productivity in order to obtain a higher skill adjustment rate that allows the individual to be more flexible in learning different skills . This not only increases future productivity and reduces future mismatch cost, but also increases the average mobility and flexibility of the labor market by increasing the ability of individuals to adapt more easily to different skill requirements. In fact, the simulation shows an increase in job mobility and the times in which the salary is renegotiated in case the optimal control is chosen. This is due to the fact that, when the cognitive skill is less trained, less skilled individuals can refuse offers from firms with high requirements because the initial mismatch cost is too high and the time needed to fill it is too much. When the cognitive skill is higher also for this group of individuals, the time required to fill the skill mismatch decreases and therefore offered by firms with high skill requirements are more advantageous even for workers with medium-low cognitive skills. I expect this dynamic to occur to a greater extent if more dimensions are introduced for applied skills. In this case, even individuals with higher cognitive skills could reject potentially more productive companies for the initial skill mismatch, and therefore an increase in the rate of skill adjustment should increase mobility for these individuals as well. Due to Bertrand competition between firms, higher average value from outside offers means higher wage value (the share of value, W ,given to workers). Since the number of times wages are renegotiated, or jobs are changed, increase with optimal control, average wages grow faster.

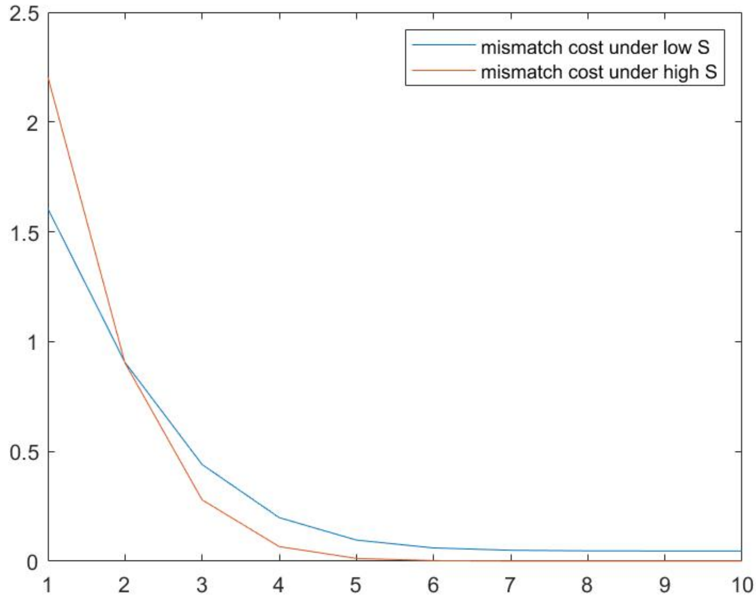

Figure 4: Average mismatch cost falls faster thanks to the higher rate of skill adjustment

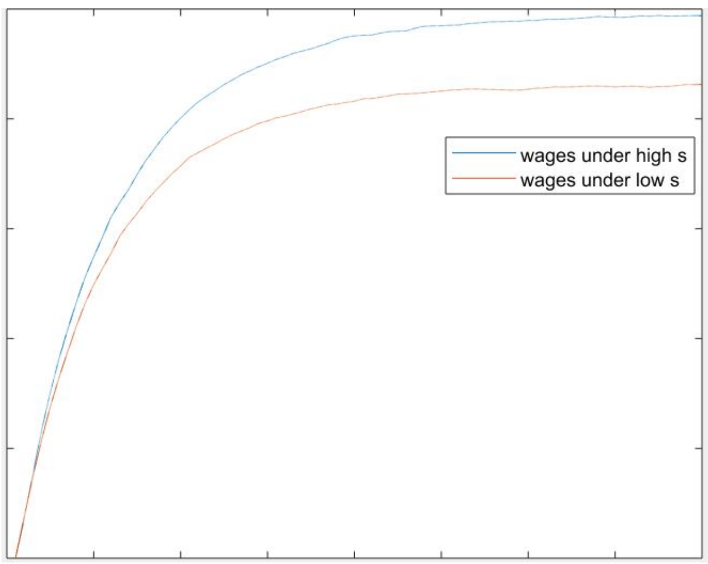

Figure 5: Average wage grows faster due to increased job mobility

Before we argued that a higher emphasis on cognitive skills in educational curricula would yield more substantial benefits for individuals who are categorized as low to medium-skilled, as opposed to those who are considered high-skilled. For low and medium-skilled individuals, an increased focus on cognitive skills amplifies their ability to swiftly adapt to new job roles and requirements. This, in turn, boosts their 'applied skill level,' a metric we define as the effective level of skill an individual can apply in their current role. The reason being that these individuals are more likely to find jobs at firms with an overall higher average skill requirement due to their enhanced adaptability. Therefore, they find themselves in environments that allow for greater growth, and consequently, their average skill level rises.

However, for high-skilled individuals, the landscape is a bit more nuanced. While these individuals are indeed smart and adaptable, they face diminishing marginal returns on intelligence in the context of learning and skill adaptation. This results in a curious dynamic; while high-skilled individuals can adapt their skills quickly, they stand to lose more due to a higher average 'initial skill mismatch' when starting new roles. Essentially, the gains from higher adaptability do not necessarily offset the losses incurred from this initial mismatch.

The upshot of this shift towards cognitive skills is evident wage dynamic, particularly under a policy optimized by a theoretical planner aiming for maximum societal benefit. Under such an optimized policy, average wages grow faster due to increased job mobility for low and medium-skilled individuals, facilitated by their enhanced adaptability. In effect, these individuals can more readily switch between roles and firms, seeking out opportunities for higher wages and thereby contributing to a wage distribution that is more tightly clustered around a higher average. It is important to remark that interpreting cognitive skills as technology in learning, and therefore not including them directly in production and in firms requirements and therefore not allowing for on-the-job training for cognitive skills is not only consistent with the empirical findings, as previously discussed, but does not impact the core result: allowing cognitive skills to participate directly in production, in fact, will make these skills even more valuable enhancing the shift of the optimal policy towards higher training of cognitive skills during education.

5 Conclusion

This study aimed to present a framework for analyzing the job market implications of an educational system that places greater relative importance on the training of cognitive skills. The educational system is modeled as a transformation of initial cognitive skills in the population into a two dimensional skill bundle composed by cognitive skills after education and applied skills acquired during education and applicable in production (marketable knowledge). Using an equation for skill accumulation in the school system - calibrated using PIAAC dataset- I identified a frontier of the possible skill distributions of workers. Each point in the frontier is a different two-dimensional skill distribution that corresponds to a different design of the educational system chosen by the theoretical planner, that chooses the stock of cognitive skills allocated to the accumulation of applied or new cognitive skills. The environment is similar to common frameworks such as Postel-Vinay and Lise (2020) and Postel–Vinay and Robin (2002) in which individuals are randomly matched with a firm and produce an instantaneous flow of output which is proportional to the workers' skill and the firm technology but has a penalty term due to skill mismatch. Workers gradually reduce the skill mismatch by adapting their skills to firms requirements.
The main difference with respect to similar models is that the rate at which they adjust their skills depends on the cognitive skill, so it is endogenous and depends on the design of the educational system. The theoretical job search model is therefore used in this first basic framework to define internal efficiency within the educational system, to formalize which dynamics can change in the latter by implementing different educational models. In particular, with the model I intend to formally explain how sacrificing learning of applied concepts or tools in exchange for greater mental elasticity can allow to extract greater value from the labor market through increased flexibility and mobility, together with a more egalitarian range of opportunities and a reduction of job polarisation, due to diminishing returns of cognitive skills in skill accumulation and the capability of cognitive skills of reducing the impact of initially larger mismatch.

The results showed that increasing the weight of cognitive skills in education results in lower initial productivity but higher rates of adjustment, particularly for low and medium skilled individuals, leading to increased overall mobility. The optimal solution was found to have a cognitive skills weight in education higher than the current level. Improving overall cognitive skills, especially for low and medium skilled individuals (diminishing marginal returns) allows workers to adjust faster to higher mismatches, and therefore allows also medium skilled workers to arrive at top jobs.

Overall, the findings of this study suggest that increasing the attention to the training of cognitive abilities in the primary school system can help create a more mobile, flexible and democratic labor market. This is particularly relevant in the context of disruptive technological innovations, which will make cognitive skills much more valuable, particularly for those who were less trained in this area during their education.
The model presents several limitations: the basic form of the production function is suitable for a close form solution which allows to make comparative statics with the explicit match value, but is not calibrated with actual data and its functional form might not fully capture real-world complexities. Moreover, the dynamic of skill acquisition during education is modeled with a general Cobb-Douglas form whose parameter are calibrated matching the workers skill distribution in the PIAAC data with the skill frontier implied by the accumulation equations. Even if in this case the modeling is informed by real data, a general form for the skill accumulation functions does not include all education dynamics. Future iterations of this model will aim to include a more nuanced representation of 'applied skills' and will consider a skills-based firm

distribution to provide a comprehensive analysis of labor market mobility and skill mismatch costs. Another important implication of this framework concerns adaptability of workers to technological innovations. An increase in the rate of skill adjustment due to a greater cognitive skill not only allows workers to adapt faster to the skills required by firms, but also to technological changes. Another important development of this work is therefore to analyze the response of the labor market to various shocks in the composition of the requirements of the firm due to disruptive technological innovations. In this environment, major technological changes will make cognitive skills even more valuable, enhancing the most gifted individuals especially if the cognitive skill is not trained in a relevant way during education. The mechanism presented in the model -in which higher cognitive skill training is able to reduce skill polarization- is expected to amplify in this context.

The conclusions drawn from this study carry significant policy implications, especially when considering the evolving landscape of the labor market due to technology-driven changes. The advent of disruptive technologies such as artificial intelligence, machine learning, and automation has already begun to reshape skill requirements across various sectors. In such a fluid context, the ability to rapidly adapt and acquire new skills becomes increasingly critical. Therefore, an educational foundation that empowers individuals with strong cognitive skills is more than just beneficial; it becomes a vital prerequisite for economic resilience and social mobility in the 21st century.

By cultivating these cognitive abilities from an early age, the education system can act as a proactive force, not just preparing students for existing jobs but for roles that have yet to be created. This forward-thinking approach ensures that future generations are not only capable of adapting to new job environments but are also better prepared to become innovators and leaders in fields that may not yet exist. This results in a workforce that is not only more flexible and adaptable but also more creative and forward-thinking, attributes that are essential for driving innovation and staying competitive in a global market. The elevated significance of cognitive skills, thus, has a cascading effect that extends far beyond individual benefits. It reaches into societal structures, potentially reducing inequalities by providing equitable access to opportunities, and even has implications for national economies, making them more agile and better prepared to adapt to global challenges. As we look towards the future, extending this model can provide more granular insights that could guide educational policy, preparing us for a world that is not just rapidly changing, but also increasingly unpredictable.

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7 Disclosure Statement

The author reports there are no competing interests to declare.

8 Data Availability Statement

The data that support the findings of this study are openly available in “figshare” at https://doi.org/10.6084/m9.figshare.22086353.v1.

References

Acemoglu, D., & Autor, D. (2011, January). Chapter 12 - Skills, Tasks and Technologies: Implications for Employment and Earnings**We thank Amir Kermani for outstanding research assistance and Melanie Wasserman for persistent, meticulous and ingenious work on all aspects of the chapter. We are indebted to Arnaud Costinot for insightful comments and suggestions. Autor acknowledges support from the National Science Foundation (CAREER award SES-0239538). In D. Card & O. Ashenfelter (Eds.), *Handbook of Labor Economics* (Vol. 4, pp. 1043–1171). Elsevier. Retrieved 2022-08-22, from https://www.sciencedirect.com/science/article/pii/S0169721811024105 doi: 10.1016/S0169-7218(11)02410-5

Allen, J., & van der Velden, R. (2007). Transitions From Higher Education to Work. In (pp. 55–78). doi: 10.1007/978-1-4020-5926-1_4

Becker, G. S. (1964). *Human Capital: A Theoretical and Empirical Analysis, with Special Reference to Education* (SSRN Scholarly Paper No. ID 1496221). Rochester, NY: Social Science Research Network. Retrieved 2021-01-15, from https://papers.ssrn.com/abstract=1496221

Carpenter, P., Just, M., & Shell, P. (1990, 08). What one intelligence test measures: A theoretical account of the processing in the raven progressive matrices test. *Psychological review*, *97*, 404-31. doi: 10.1037/0033-295X.97.3.404

Coleman, J. S. (1968). Equality of educational opportunity. *Integrated education*, *6*(5), 19–28. (Publisher: Taylor & Francis)

Commander, S., & Kollo, J. (2004). *lThe Changing Demand for Skills: Evidence from the Tran $ sition* (Tech. Rep.). mIZA Discussion Paper.

Dachs, B. (2018). *The impact of new technologies on the labour market and the social economy*. European Parliamentary Research Service.

Dearden, L., McIntosh, S., Myck, M., & Vignoles, A. (2002). The Returns to Academic and Vocational Qualifications in Britain. *Bulletin of Economic Research*, *54*(3), 249–274. Retrieved 2021-05-14, from https://onlinelibrary.wiley.com/doi/abs/10.1111/1467-8586.00152 doi: https://doi.org/10.1111/1467-8586.00152

de Chastelaine, M., Wang, T. H., Minton, B., Muftuler, L. T., & Rugg, M. D. (2011). The effects of age, memory performance, and callosal integrity on the neural correlates of successful associative encoding. *Cerebral Cortex*, *21*(9), 2166–2176. (Publisher: Oxford University Press)

Flinn, C., Gemici, A., & Laufer, S. (2017). Search, matching and training. *Review of Economic Dynamics*, *25*, 260–297.

Gathercole, S. E., Dunning, D. L., Holmes, J., & Norris, D. (2019, April). Working memory training involves learning new skills. *Journal of Memory and Language*, *105*, 19–42. Retrieved 2022-08-22, from https://www.sciencedirect.com/science/article/pii/S0749596X18300871 doi: 10.1016/j.jml.2018.10.003

Guo, S. (2010, 03). Book review: Jarvis, p. (2006). the lifelong learning and the learning society trilogy, volumes 1-3. *Adult Education Quarterly - ADULT EDUC QUART*, *60*, 207-211. doi: 10.1177/0741713609350488

Guvenen, F., Kuruscu, B., Tanaka, S., & Wiczer, D. (2020). Multidimensional skill mismatch. *American Economic Journal: Macroeconomics*, *12*(1), 210–244.

Hanushek, E. A., Schwerdt, G., Woessmann, L., & Zhang, L. (2017). General education, vocational education, and labor-market outcomes over the lifecycle. *Journal of Human Resources*, *52*(1), 48–87.

Heckman, J. J. (2007). The economics, technology, and neuroscience of human capability formation. *PNAS*, *104*(33), 13250–13255. Retrieved from https://www.pnas.org/content/104/33/13250 doi: 10.1073/pnas.0701362104

Hegarty, M. (2004, 06). Dynamic visualizations and learning: Getting to the difficult questions. *Learning and Instruction*, *14*, 343-351. doi: 10.1016/j.learninstruc.2004.06.007

Illanes, P., Lund, S., Mourshed, M., Rutherford, S., & Tyreman, M. (2018). Retraining and reskilling workers in the age of automation. *McKinsey Global Institute*.

Ismail, N., & Elias, S. (2006, 06). Inquiry-based learning: An innovative teaching method. *UiTMJ Academic Journal of Social Studies. Vol.6 (2): 35-44 ISSN 1511-9300*.

Jensen, A. R. (1989, January). The relationship between learning and intelligence. *Learning and Individual Differences*, *1*(1), 37–62. Retrieved 2021-01-18, from http://www.sciencedirect.com/science/article/pii/1041608089900095 doi: 10.1016/1041-6080(89)90009-5

Katz, L. F., & Autor, D. H. (1999, January). Chapter 26 - Changes in the Wage Structure and Earnings Inequality. In O. C. Ashenfelter & D. Card (Eds.), *Handbook of Labor Economics* (Vol. 3, pp. 1463–1555). Elsevier. Retrieved 2022-08-22, from https://www.sciencedirect.com/science/article/pii/S1573446399030072 doi: 10.1016/S1573-4463(99)03007-2

Lamo, A., Messina, J., & Wasmer, E. (2011). Are specific skills an obstacle to labor market adjustment? *Labour Economics*, *18*(2), 240–256.

Lindenlaub, I. (2014). Essays on heterogeneity in labor markets.

Lindenlaub, I. (2017, April). Sorting Multidimensional Types: Theory and Application. *The Review of Economic Studies*, *84*(2), 718–789. Retrieved 2022-08-22, from https://doi.org/10.1093/restud/rdw063 doi: 10.1093/restud/rdw063

Lindenlaub, I., & Postel-Vinay, F. (2016). Multidimensional sorting under random search. *Manuscript*, *University College London*, *15*.

Lucas, R. E. (1988). On the mechanics of economic development. *Journal of Monetary Economics*, *22*(1), 3–42.

Machin, S. (2003). Wage inequality since 1975. In *The labour market under new labour* (pp. 191–200). Springer.

Machin, S., & Reenen, J. (1998). Technology and Changes in Skill Structure: Evidence From Seven OECD Countries. *The Quarterly Journal of Economics*, *113*, 1215–1244. doi: 10.1162/003355398555883

McIntosh, S. (2002). The changing demand for skills. *European journal of education*, *37*(3), 229–242. (Publisher: JSTOR)

Morrison, J. H., & Hof, P. R. (1997). Life and death of neurons in the aging brain. *Science*, *278*(5337), 412–419. (Publisher: American Association for the Advancement of Science)

Panth, B., & Maclean, R. (2020). Introductory overview: Anticipating and preparing for emerging skills and jobs—issues, concerns, and prospects. *Anticipating and Preparing for Emerging Skills and Jobs*, 1–10. (Publisher: Springer, Singapore)

Persson, J., Nyberg, L., Lind, J., Larsson, A., Nilsson, L.-G., Ingvar, M., & Buckner, R. L. (2006). Structure–function correlates of cognitive decline in aging. *Cerebral cortex*, *16*(7), 907–915. (Publisher: Oxford University Press)

Postel-Vinay, F., & Lise, J. (2020). Multidimensional skills, sorting, and human capital accumulation. *The American Economic Review*, *110*(8), 2328–2376. (Publisher: American Economic Association)

Postel–Vinay, F., & Robin, J. (2002). Equilibrium wage dispersion with worker and employer heterogeneity. *Econometrica*, *70*(6), 2295–2350. (Publisher: Wiley Online Library)

Redecker, C., Leis, M., Leendertse, M., Punie, Y., Gijsbers, G., Kirschner, P., ... Hoogveld, B. (2010). The future of learning: New ways to learn new skills for future jobs. *Results from an online expert consultation. Technical Note JRC60869, JRC-IPTS, Seville.*
Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, *98*(5), S71–S102.
Rosen, S. (1974, January). Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition. *Journal of Political Economy*, *82*(1), 34–55. Retrieved 2022-08-22, from https://www.journals.uchicago.edu/doi/10.1086/260169 (Publisher: The University of Chicago Press) doi: 10.1086/260169
Salat, D. H., Buckner, R. L., Snyder, A. Z., Greve, D. N., Desikan, R. S., Busa, E., ... Fischl, B. (2004). Thinning of the cerebral cortex in aging. *Cerebral cortex*, *14*(7), 721–730. (Publisher: Oxford University Press)
Sanders, C. (2012, 1). Skill uncertainty, skill accumulation, and occupational choice. *2012 Meeting Papers*. Retrieved from https://ideas.repec.org/p/red/sed012/633.html
Sanders, C., & Taber, C. (2012). Appendix for: Lifecycle Wage Growth and Heterogeneous Human Capital.
Schultz, T. W. (1961). Investment in Human Capital. *The American Economic Review*, *51*(1), 1–17. Retrieved 2021-05-14, from https://www.jstor.org/stable/1818907
Taber, C., & Vejlin, R. (2016). *Estimation of a Roy/search/compensating differential model of the labor market* (Tech. Rep.). National Bureau of Economic Research.
Tinbergen, N. (1956). On the functions of territory in gulls. *Ibis*, *98*(3), 401–411. (Publisher: Wiley Online Library)
Vaci, N., Edelsbrunner, P., Stern, E., Neubauer, A., Bilalić, M., & Grabner, R. H. (2019, September). The joint influence of intelligence and practice on skill development throughout the life span. *PNAS*, *116*(37), 18363–18369. Retrieved 2021-01-15, from https://www.pnas.org/content/116/37/18363 doi: 10.1073/pnas.1819086116

9 Appendix

In this section I will derive the solution for the closed form solution (18) of the PDE

$$\beta P(x,y) = p(x,y) - \sigma P(x,y) + \nabla_x P(x,y)\dot{X}(x,y) \tag{21}$$

We will consider the relevant case in which the worker is underqualified with respect to both skills. computing the scalar product in (23) and rearranging we get:

$$(\beta+\mu)P = p(x,y) + \frac{dP}{dx_A} x_C^{\alpha}(y_A - x_A) \tag{22}$$

By similarity, we will search for a solution in the following form:

$$P(x,y) = f(x_A, y_A) - \frac{k_A(y_A - x_A)^2}{\beta+\mu+x_C^{\alpha}} \tag{23}$$

plugging into the PDE (24) we obtain:

$$f(x_A,y_A)(\beta+\mu) - \frac{(\beta+\mu)k_A(y_A-x_A)^2}{\beta+\mu+x_c^{\gamma}} = x_A y_A - k_A(y_A-x_A)^2 + f^{'}(x_A,y_A)(x_c^{\gamma})(y_A-x_A) - 2\frac{(\beta+\mu)k_A x_c^{\gamma}(y_A-x_A)^2}{\beta+\mu+x_c^{\gamma}} \tag{24}$$

Computing the above, all left terms simplifies and we are left with;

$$f(x_A,y_A) = f^{'}(x_A,y_A)(x_c^{\gamma})(y_A - x_A) + x_A y_A \tag{25}$$

so that the PDE is reduced to a standard ODE. Using Duhamel's formula we obtain the solution for f:

$$f(x_A,y_A) = y_A(x_A + x_c^{\gamma}(y_A - x_A)) \tag{26}$$